# Impulsive Loading on Reinforced Concrete Slabs - Modelling considerations 

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#### Abstract

Structural modelling has been widely and successfully used for assessing the characteristics and behaviour of materials and structures produced at different scales. There is great economic and practical advantage in using smaller structures exposed to scaled loads that represent the real structure and loads. Additionally, when tests require the use of explosives as in this research, structural modelling provides the powerful tool for avoiding use of greater amounts of explosive, so eliminating unnecessary risks. A structural model is usually described as structural element or assembly of structural elements built to a reduced scale (in comparison with full size structures) which is to be tested, and for which laws of similitude must be employed to interpret test results.

This paper describes "gravity neglected - adequate model" method used to study structural response of RC slabs exposed to close range explosion. Blast parameters were modelled using cube root scaling laws. The results of experiments at two different scales are presented and comparison made between them. Recorded deflections, local and overall damage and crack patterns have been examined.


## Introduction

The similitude requirements that relate a model to the real structure are based upon the theory of modelling which is derived from a dimensional analysis of the physical phenomena involved in the behaviour of the structure. The dimensional characteristics widely used for describing the physical phenomena are:
(a) Length
(b)
Force (or mass)
(c) Time
(d)

Temperature (e) Electric charge
Since most of the structural problems are of mechanical nature the first three above mentioned dimensions are the most important for structural engineering. The main requirement of dimensional analysis is that any mathematical relationship that describes the behaviour of a structure must be dimensionally valid regardless of the parameters that quantify the effects. This implies that any relation of the form $F\left(X_{1}, X_{2} \ldots \ldots X_{n}\right)=0$ can be expressed in term of $G\left(\pi_{1}, \pi_{2} \ldots \ldots . \pi_{\mathrm{m}}\right)=0$ where the $\pi^{\prime}$ s are dimensionless measures of physical effects previously given in the form of $X_{1}$ to $X_{n}$. This allows a reduction of the unknown quantities $m$ that fully represent physical behaviour of the structure, because $m=n-r$, where $r$ is the number of fundamental dimensions that are involved in the physical phenomena. This means that a dynamic problem that combines the effects of say 6 different parameters, can be effectively reduced, in our case, to three dimensionless parameters, because $\mathrm{r}=3$ (length, time and mass).

Dimensional analysis and structural modelling can use replica models of complete similarity with the prototype when all of the dimensionless products are exactly the same in both model and prototype, or adequate models that provide a close similarity but eliminate those variables that are not considered of relevant importance. Since it is usually very difficult to obtain exact similitude replica modelling, adequate models are used most frequently.

## Theory of modelling for structures exposed to impact and blast loading

Modelling considerations for transient dynamic loading include the loading function (force, pressure, time, gravity and velocity) the geometry of the specimen (linear dimensions, displacement and strain) and the material characteristics (modulus of elasticity, stress, Poisson's ratio, mass and mass density).

The approach that would provide a so called true replica model would require selection of three physical quantities for independent scaling since there are three independent basic quantities (M, L, T) that describe the phenomena, Sabnis et al [1], Noor and Boswell [2]. Since in all possible combinations the gravity acceleration must be the same for the model and the prototype, two additional quantities can be chosen as say Poisson's ratio and velocity V . To fulfil the dimensional analysis requirements, time, linear dimensions and displacement would need to be scaled with a linear scaling factor. Strain, gravity acceleration, Poisson's ratio and
velocity would be the same for the model and the prototype, but scaling of mass density, mass, modulus of elasticity, force and pressure would require an additional change of modulus of elasticity of the material. This inevitably leads to the use of different material than concrete which is often not acceptable and consequently this true replica model can not be of great use in dynamic modelling of structures.

The model that one would naturally be expected to use would require the same material characteristics in the model and prototype and a linear scaling of geometry.

An adequate model which would provide these requirements is called a Gravity Forces Neglected model and has been widely used for dynamic modelling and is also used in this research. The main difference with a true replica model is that gravitational acceleration $g$ is neglected. This approach is acceptable since gravitation forces do not represent a significant part of the loading function in the cases of impact and blast loading. The relationship of the physical quantities for model specimens and those of the real structure, the full scale specimens, used in this research are given in Table 1.

Table 1. Summary of scale factors for dynamic loading

| $\begin{array}{c}\text { PHYSICAL } \\ \text { QUANTITIES }\end{array}$ | DIMENSION |
| :--- | :---: | :---: | \(\left.\begin{array}{c}ADEQUATE <br>

MODEL\end{array}\right]\)
$\mathrm{M}, \mathrm{L}$ and T represent units for mass, length and time respectively, Sl is the linear scaling factor between the model and the prototype and 1 means that values are the same in both scales.

## Blast wave scaling and parameters

The scaling law for explosions is based on geometrical similarity. The explosive charges and distances from the specimen were scaled according to the cube root scaling laws. Since the TNT equivalent has been widely accepted as a measure of the characteristics of different explosives, a spherical charge of conventional chemical explosive with energy release equivalent to one kilogram TNT will be used to explain the principle of blast scaling.

The scaling law for explosions is based on conservation of momentum and geometrical similarity. Geometrical similarity of three dimensional bodies such as an explosive charge leads to a third power of ratio relations which are often represented in most blast wave scaling applications.

Two explosions can be expected to produce identical blast waves at distances which are proportional to the cube root of the respective energy release which is taken as the controlling parameter. Conservation of momentum can be introduced through the density of the atmosphere as a measure of the mass of the air which leads to the expression for the scaled distances $Z$ as:

$$
Z=\frac{f_{d} \cdot(\text { actual distance })}{\sqrt[3]{W}}
$$

where: $f_{d}$ - dimensionless ratio of the density of the atmosphere through which an explosive shock travels and that of the atmosphere for the reference explosion.
$W$ - explosion yield (kg of TNT)
If $f_{d}$ is taken as one, it can be shown that two charges of the same shape and explosive type but different masses $M_{1}$ and $M_{2}$ have peak overpressures that occur at distances that are related as:

$$
R_{1}=k \cdot R_{2} \quad \text { where }: \quad k=\sqrt[3]{\frac{M_{1}}{M_{2}}}
$$

Although the peak overpressures will be the same at $R_{1}$ and $R_{2}$, the scaling of time will mean that the other important parameters as duration of the pulse $T_{d}$ and its impulse $I$ are not the same and they can also be given as:

$$
I_{1}=k \cdot I_{2} \quad \text { and } \quad T d_{1}=k \cdot T d_{2}
$$

If non uniform atmospheres are considered $f_{d}$ cannot be taken as one and the above mentioned relations have to be adjusted for temperature and atmospheric density factors.

The explosive used in these tests was plastic explosive PE4 which had mass density of $1590 \mathrm{~kg} / \mathrm{m}^{3}$, detonation velocity of $8189 \mathrm{~m} / \mathrm{sec}$, detonation pressure of $2.68 \times 10^{7} \mathrm{kN} / \mathrm{m}^{2}$ and mass specific energy of $5111 \mathrm{~kJ} / \mathrm{kg}^{2}$, which gives it a TNT equivalent of 1.13.

Apart from a few initial tests on the small scale slabs where the charge was cylindrical in shape, all charges were of hemispherical shape with the spherical side of the charge facing the specimen.

They were all hand made from 454 g explosive sticks that were compacted in to the specially made plastic moulds, so producing a charge of uniform shape and consistent density. The L2A1 detonators were placed into a pre-formed 10 mm deep hole in the centre of the flat side of the charge in all tests, and then held in place by insulation tape.

In all tests the charges were initiated from the side furthest from the specimen. The large scale charge was chosen to be 1300 g since the large blast cell has been blast - proved for that amount of explosive. The diameter of the hemispherical charge was 142.5 mm . Detailed description of the tests is given by Duranovic and Watson, [3] and [4].

Cube root scaling indicates that a charge of mass $\mathrm{M}_{1}=1300 \mathrm{~g}$ will produce the same peak overpressure and shock wave velocity at a distance $R_{1}$ from the charge, as a scaled charge of mass $M_{2}$ of the same explosive type and shape at range $R_{2}$ when: $\frac{\mathrm{R}_{1}}{\sqrt[3]{\mathrm{M}_{1}}}=\frac{\mathrm{R}_{2}}{\sqrt[3]{\mathrm{M}_{2}}}$ So the scale factor is: $\frac{R_{1}}{R_{2}}=\sqrt[3]{\frac{M_{1}}{M_{2}}}$ and for: $\frac{R_{1}}{R_{2}}=2.5$ and $M_{1}=1300 \mathrm{~g}$ we obtained $M_{2}=\frac{1300}{2.5^{3}}=83 \mathrm{~g}$. For practical reasons (the same size detonator was used for both scales), the model charge was actually $\mathrm{M}_{2}=78 \mathrm{~g}$ and it had a diameter of 57 mm .

Although the scaled charges gave the same peak pressure and shock wave velocity at scaled distances, the positive duration and impulse produced by the larger charge were 2.5 times greater than corresponding values produced by the smaller charge at scaled distances.

## Experimental technique and test results

The reinforced concrete slab specimens used in this research are based on typical structural elements which can exist in various types of structure, for example industrial buildings. The models have been designed to represent approximately $1: 4$ scale and $1: 10$ scale models of typical prototypes. Thus the small specimens (SE) model the large specimens (LSE) at 1:2.5 scale. Supports were fully fixed in all cases.

The concrete mix used for the production of the model slabs contained river sand, max. size 4 mm ; $\mathrm{W} / \mathrm{C}$ ratio was 0.6 and the aggregate/cement ratio was 2.28 . The average static compressive strength was $40 \mathrm{~N} / \mathrm{mm}^{2}$. Typical small scale slab is presented in Figure 1. Further test details are given in Table 2

top layer reinforcement:
R.MESH: 3.15 mm DIAM./76.2 mm CENTRES
(without central region $400 \times 400 \mathrm{~mm}$ )
bottom layer reinforcement:
R.MESH: 3.15 mm DIAM./76.2 mm CENTRES
cover: 4 mm
Figure 1 Model slab geometry
Table 2. Test details

| Slab | Standoff | Tensile <br> No: <br> distance <br> $(\mathrm{mm})$ |  |
| :---: | :---: | :---: | :---: |
|  | reinforcement |  |  |


| Slab <br> No: | Standoff <br> distance <br> $(\mathrm{mm})$ | Tensile <br> reinforcement |  |
| :---: | :---: | :---: | :---: |
|  |  | Y way |  |
| SE12 | 150 | $0.29 \%$ | $0.27 \%$ |
| SE5 | 100 | $1.25 \%$ | $1.05 \%$ |
| SE11 | 200 | $0.29 \%$ | $0.27 \%$ |
| SE15 | 75 | $0.29 \%$ | $0.27 \%$ |

In the scaling laws the strain-rate effects and gravity effects are assumed negligible. This section provides a comparison between damage and displacements on the full and small scale slabs. Table 3 gives a comparison between some of the results.

Table 3. Blast Impulse Tests, displacement scaling
BLAST IMPULSE TESTS

| :1 SCALE RC SLABS | 1:2 | (\%) |
| :---: | :---: | :---: |
| LSE1 <br> Charge standoff: 350mm Measured displacements: at 225 mm off centre: 26.3 mm , at 450 mm off centre: 18.8 mm . (These positions correspond to 90 and 180 mm off centre on 1:2.5 scale slabs, respectively) | Scaled standoff distance : 140 mm SE12 - charge standoff: 150mm : Measured displacements : at 80 mm off centre : $\sim 21 \mathrm{~mm}$ at 160 mm off centre: $\sim 13 \mathrm{~mm}$ at 240 mm off centre : $\sim 7 \mathrm{~mm}$ Estimated disp. at $90 \mathrm{~mm}: \sim 20 \mathrm{~mm}$ Estim. disp. at $180 \mathrm{~mm}: \sim 11.5 \mathrm{~mm}$ |  |
| Reinforcement ratio of slab SE12 was significantly smaller than on LSE1 and the small slab had the top reinforcement discontinued at the centre. Although the standoff distance used in the test was only slightly smaller than required the results cannot be directly compared. |  |  |
| LSE2 <br> Charge stand-off: 250mm Measured displacements: at 225 mm off centre: 31.9 mm at 450 mm off centre: 20 mm | Scaled stand-off distance : 100 mm SE5 - charge stand-off: 100mm Measured displacement: <br> at 180 mm off centre: $\quad 6.3 \mathrm{~mm}$ | \% |
| $\begin{array}{\|lc\|}\text { Scaled displacement at } 180 \mathrm{~mm} \text { off centre: } & 6.3 \times 2.5=15.75 \mathrm{~mm} \\ \text { Relative difference in dis. between two scales: } & \text { R.D. }=\frac{20-15.75}{20}=0.212\end{array}$ |  |  |
| LSE3 <br> Charge stand-off $=500 \mathrm{~mm}$ <br> Measured displacements: <br> at the centre: $\quad 52.2 \mathrm{~mm}$ <br> at 100 mm off centre: 45.2 mm <br> at 200 mm off centre: 38.9 mm <br> at 300 mm off centre: 33.5 mm | Scaled stand-off distance : 200mm <br> SE11 - charge stand-off: 200mm Measured displacements: <br> at 80 mm off centre : $\sim 19 \mathrm{~mm}$ <br> Est. disp. 120mm of cen.: $\sim 15 \mathrm{~mm}$ | $\begin{aligned} & 22.1 \% \\ & 11.9 \% \\ & \hline \end{aligned}$ |

The positions at which meas. were taken on the large scale slab, correspond to: centre, 40, 80, and 120 mm off centre on $1: 2.5$ scale slabs, respecitvely

| LSE5 | Scaled stand-off distance: 80 mm |  |
| :---: | :---: | :---: |
| Charge stand-off $=200 \mathrm{~mm}$ | SE15 - charge stand-off: 75mm |  |
| Measured displacements: | Measured displacements: <br> at 80 mm off centre : $\sim 18 \mathrm{~mm}$ |  |
| at 300 mm off centre: 38.2 mm | Estim. disp. at 120mm: $\sim 15.6 \mathrm{~mm}$ | 2.1\% |
| at 400 mm off centre: 32.7 mm | Estim. disp. at 160mm: $\sim 13,2 \mathrm{~mm}$ | 0.9\% |
|  | at 180mm : $\quad \sim 12 \mathrm{~mm}$ |  |
| at 500 mm off centre: $\mathbf{2 3 . 4 m m}$ | Esti. disp. at 200 mm : $\sim 9.8 \mathrm{~mm}$ | 4.7\% |

The positions at which meas. were taken on the large scale slab, correspond to: $120 \mathrm{~mm}, 160 \mathrm{~mm}$ and 200 mm off centre on $1: 2.5$ scale slabs, respectively.

In Table 3 the Relative Difference in results (R.D.), incorporates the error in scaling between two scales and also usual scatter in test results, which always appears in testing of reinforced concrete elements.

Estimated values have been obtained by either the means of linear interpolation or linear extrapolation.

The results show that when overall flexural response was dominant, as in slabs LSE5 and SE15, there was much better scaling then in cases when local response was dominant.

Local damage is quantitatively very similar at both scales. The same pattern of spall and scab damage was produced and the quantifiable damage on slabs of two different scales compare well.

In both cases the spalls are very minor and the scabs are extensive and the same kind of circumferential and radial cracking around the epicentre indicates similar failure mechanisms. The similarity of loading and cross section characteristics allows comparison between the damage on the large slab LSE5 and small slab SE15, both subjected to explosive blast. Neither of them was perforated, the spall was slightly larger on LSE5 and the scab percentages are within $1.4 \%$. The top and bottom face local cracking is almost identical. Slab SE16 had a same charge as SE15, only 15 mm closer to the slab but produced a different failure mechanism and perforation.

Yield line patterns dominate the shape of flexurally produced damage at both scales. The same types of cracks appear on slab surfaces at both scales, indicating the existence of identical response patterns at both scales, Fig. 2.


Figure 2. Crack patterns for two slabs of different scales (LSE5 and SE15)
It can be seen from the previous Figure that both local and overall damage were almost identical despite the fact that slab on the right is 2.5 times smaller than the left one. Identical local responses under the explosive charge (central region) indicate that it is possible to model even
the scab formation and crack development in the early stages of R.C. slab response, Duranovic [5].

It is important to notice that scaling of local and overall damage of the slab was even more successful then scaling of slab displacements or reinforcement strains.

## Conclusions

The following conclusions can be drawn from the work described in this paper:
(a) Manufacturing and testing problems together with the cost of full scale specimens make the use of model specimens extremely beneficial providing the results and behaviour can be compared to real structures.
(b) The modelling laws employed in this research can successfully be used for structural dynamic modelling. By neglecting the existence of gravitational forces results can not be much affected but the neglect of strain rate effects is more important and can affect the test results.
(c) The displacements obtained on small scale specimens were expected to be 2.5 times smaller than on the full scale specimens for the scaled loading and support conditions. Results from some of the related specimens are shown in Table 3. The ratios of displacement on 1:1 scale slabs and on 1:2.5 scale slabs magnified by 2.5 , vary between $77.9 \%$ and $99.1 \%$.
(d) The local and overall damage was almost identical in appearance for both sizes of slab which indicates the same failure mechanism for real structure and the scaled model.

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